Dynamic Fluctuation Phenomena in Double Membrane Films

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Dynamics of double membrane films is investigated in the long-wavelength limit $qh \ll 1$ (q is the wave vector and h is the thickness of the film) including the over damped squeezing mode. We demonstrate that thermal fluctuations essentially modify the character of the mode due to its nonlinear coupling to the transversal shear hydrodynamic mode. The renormalization can be analysed under condition $g \ll 1$ (where $g \sim T/\kappa$, T is the temperature and κ is the bending module). The corresponding Green function acquires as a function of the frequency ω a cut along the imaginary semi-axis. At $qh > \sqrt{g}$ the effective length of the cut is $\sim Tq^3/\eta$ (where η is the shear viscosity of the liquid). At $qh < \sqrt{g}$ fluctuations lead to increasing the attenuation of the squeezing mode: it is larger than the 'bare' value by the factor $1/\sqrt{g}$.

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I. INTRODUCTION

The most distinctive property of amphiphilic molecules is their ability to spontaneously self-assemble into aggregates of various shapes. Typically the molecules spontaneously self-assemble into membranes which are bilayers of a thickness of the order of a molecular length. Different lyotropic structures constituted of the membranes have generated considerable current interest (see the books [1–3] and the reviews [4–6]). Films composed of two bilayer membranes sandwiching a thin layer of a liquid are widely spread in the lyotropic systems. They play also an essential role for various biological processes (one can note the so-called flickering phenomena in erythrocytes or red blood cells). In the paper we will examine dynamic properties of such double membrane films.

The main peculiarity of a membrane is its negligible surface tension. Indeed, the membrane is immersed into a liquid and consequently its area can vary. Zero surface tension is the equilibrium condition with respect to the variations. In the situation shape fluctuations of the membrane are determined by the bending elasticity, the corresponding energy is [7,8]

$$\mathcal{H}_{\rm curv} = \frac{\kappa}{2}\int dA\,\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2\,, \tag{1.1}$$

where the integral is taken over the membrane which is considered as a two-dimensional object, R_1, R_2 are its local curvature radii, and κ is the bending rigidity module. Corrugations of the membrane induced by the thermal noise lead to loosing the orientation correlation of the membrane pieces at separations larger than the so-called persistent length ξ_p [9] which can be estimated as

$$\xi_p \sim a\exp(2\pi\kappa/T)\,,$$

where T is the temperature and a is the thickness of the membrane. The shape fluctuations of the membrane lead to the logarithmic renormalization of the bending module κ, examined first by Helfrich [10] and later by Förster [11], the correct renormalization-group equation was derived by Peliti and Liebler [12], Kleinert [13] and Polyakov [14]. The explicit form of the one-loop RG equation is

$$\frac{d\kappa}{d\xi} = -\frac{3T}{4\pi}\,.$$

Here $\xi = \ln(r/a)$ and r is the characteristic scale. As follows from the equation the role of the dimensionless coupling constant is played by the quantity

$$g = \frac{3T}{4\pi\kappa}\,. \tag{1.2}$$

Note that $\ln(\xi_p/a) \sim g^{-1}$. For real membranes $g \sim 10^{-2} - 10^{-3}$ and consequently we can treat g as a small parameter. The smallness of g means that there exists a wide range of scales $r < \xi_p$ where thermal fluctuations can be treated in the framework of the perturbation theory.

Corrugations of the membranes in a double film can be decomposed into undulation (or bending) deformations and the squeezing ones. The bending deformations are characterised by the displacement u of the film as a whole from its equilibrium position and the squeezing deformation is characterised by variations of the film thickness h (which is the separation between the membares). We will believe that in equilibrium the film lies along the $X-Y$ plane. Then in the harmonic approximation it follows from (1.1) that the energy of the film is

$$\mathcal{H} = \int dx\, dy\, \left[\kappa(\nabla^2 u)^2 + \frac{\kappa}{4}(\nabla^2 h)^2\right], \tag{1.3}$$

where both u and h are treated as functions of x and y and ∇ is the two-dimensional gradient.

At deriving (1.3) we neglected the interaction between the membranes. First, one should remember about steric interaction associated with a certain restriction of accessible configurations for one membrane in the presence of the second membrane [8]. The explicit expression for the energy is [15]

$$\mathcal{H}_{\rm ster} = \int dx\, dy\, \frac{3\pi^2 T^2}{128\kappa h^2}\,. \tag{1.4}$$

Due to the interaction (1.4) two membranes can be treated as independent only on scales smaller than $g^{-1/2}h$. Therefore (1.3) is the main contribution to the energy if

$$qh > \sqrt{g}\,, \tag{1.5}$$

where q is the characteristic wave vector. Second, we should take into account the Van der Waals interaction. We assume that the same liquid is inside and outside the film. Then the Van der Waals energy is [16]

$$\mathcal{H}_{\rm vdw} = \int dx\, dy\, \frac{H a^2}{2\pi h^4}\,, \tag{1.6}$$

where H is the Hamaker constant. We can neglect the energy in comparison with (1.3) if

$$(qh)^4 > \frac{H}{\kappa}\left(\frac{a}{h}\right)^2.$$

In the following we will believe that the thickness of the film is large enough for the inequality

$$g^2 > \frac{H}{\kappa}\left(\frac{a}{h}\right)^2,$$

to be satisfied. Then (1.5) is the only restriction enabling us to treat the energy (1.3) as the main contribution to the film energy.

II. DYNAMICS

We will examine the dynamics of the double membrane film in the long-wavelength limit $qh \ll 1$ where q is the wave vector of the eigen modes of the film. Note that the inequality $qh \ll 1$ is compatible with (1.5) since $g \ll 1$. In the limit $qh \ll 1$ one should take into account the following variables describing the dynamics: the velocity of the film $\boldsymbol{v}$, the displacement of the film u, the film thickness h and the densities of both membranes since they are conserved quantities. We will be interested mainly in the squeezing mode associated with the relaxation of the thickness h.

To find dynamical characteristics of the film one should solve the conventional hydrodynamic equations in bulk supplemented by boundary conditions on both membranes. In the linear approximation the problem was solved by Brochard and Lennon [17], they found the dispersion law of the squeezing mode

$$\omega = -i\frac{\kappa h^3 q^6}{24\eta}\,, \tag{2.1}$$

where ω is the frequency of the mode and η is the viscosity of the liquid surrounding the membranes. At deriving (2.1) one assumed that in equilibrium the film is flat. Note also the dispersion law [17]

$$\omega = -i\frac{\kappa q^3}{2\eta}\,, \tag{2.2}$$

of the bending mode also found in the linear approximation. Note that the dispersion law (2.1) is correct only if to neglect the direct interaction of the membares that is at the condition (1.5) whereas the region of applicability of the dispersion law (2.2) does not depend on the interaction of the membranes since they move in-phase in the bending mode. The elastic modes associated with variations of the membrane densities are harder than (2.1,2.2) [18]. Therefore the only effect of the elastic degrees of freedom at examining the squeezing mode is the incompressibility condition

$$\nabla_\alpha v_\alpha = 0\,. \tag{2.3}$$

Here and below we believe that all variables characterising the film are functions of x and y and assume that Greek subscripts run over x and y.

We will consider the renormalization of the dispersion law (2.1) of the squeezing mode due to fluctuational effects. For the purpose nonlinear dynamical equations of the film should be utilised. In the long-wavelength limit $qh \ll 1$ the equations can be derived phenomenologically. The reactive (non-dissipative) part of the equations can be found using the Poisson brackets method (see [19] and also [18]) whereas the dissipative part of the equations is expressed via kinetic coefficients. One should know the expression for the energy $\mathcal{H}$ of the system for writing both contributions. Actually the expression only for one Poisson bracket will be needed for us:

$$\{j_\alpha(x_1,y_1), h(x_2,y_2)\} = h(x_1,y_1)\nabla_\alpha[\delta(x_1-x_2)\delta(y_1-y_2)]\,, \tag{2.4}$$

where j_α is the two-dimensional momentum density of the film. The expression (2.4) is characteristic of any scalar conserved quantity of a film [18], the expression (2.4) is motivated by the fact that the two-dimensional mass density of the film is ρh where ρ is the three-dimensional density of the liquid. Note that $j_\alpha \approx \rho h v_\alpha$ since we believe that the membrane thickness a can be neglected in comparison with the film thickness h.

The dynamic equation for the thickness h has the standard form following from (2.4)

$$\partial_t h + \nabla_\alpha(v_\alpha h) = \Gamma\nabla^2\frac{\delta\mathcal{H}}{\delta h}\,, \tag{2.5}$$

where $\partial_t \equiv \partial/\partial t$ and Γ is the kinetic coefficient. The second power of the gradient appeared in (2.5) since the equation should support the conservation law of the liquid inside the film and therefore the right-hand side of the equation should be a full derivative at any $\mathcal{H}$. In the linear approximation we can neglect the sweeping term in (2.5) due to (2.3). Substituting the harmonic expression (1.3) for the energy $\mathcal{H}$ into (2.5) and comparing the result with (2.1) one gets

$$\Gamma = h^3/(12\eta)\,. \tag{2.6}$$

Note that Γ is inversely proportional to the shear viscosity coefficient. The point is that the dissipation described by Γ comes from viscous motion of the liquid surrounding the double membrane film which is excited hardly at large η.

The dynamic equation for j_α has the following form [21]

$$\partial_t j_\alpha - \{\mathcal{H}, j_\alpha\} = J_\alpha\,, \tag{2.7}$$

where $\boldsymbol{J}$ is the momentum flow from the bulk to the film. Just the term supplies the main dissipation of the film momentum and therefore we neglected the internal viscosity. In the linear approximation [21]

$$J_\alpha = -2\eta\hat{q}v_\alpha\,, \tag{2.8}$$

where $\hat{q}$ is the non-local operator which is reduced to multiplying by the absolute value of the wave vector q in the Fourier representation. The Poisson bracket $\{\mathcal{H}, j_\alpha\}$ can be reduced to the divergence of the symmetric stress tensor for any energy $\mathcal{H}$ [18]. Actually only the contribution associated with the Poisson bracket (2.4) and created by the harmonic energy (1.3) is relevant for us. Then the equation (2.7) is written as

$$\partial_t j_\alpha + \frac{\kappa}{2}h\nabla_\alpha\nabla^4 h = -2\eta\hat{q}v_\alpha\,. \tag{2.9}$$

We will not present here dynamical equations for the variables j_z, u and the densities of the membranes. The reason is that the equations for j_z and u describing the bending mode decouple in the approximation needed for us from the equations (2.5,2.9). Actually the equations for j_z and u are the same as for a single membrane, the corresponding nonlinear equations can be found in [18] and also in [20,21]. As to the equations for the densities of the membranes they need a separate consideration which will be presented elsewhere. The only role of the degrees of freedom at analysing the squeezing mode is reduced to the incompressibility condition (2.3).

III. RENORMALIZATION OF SQUEEZING MODE

As is seen from (2.1) in the long-wavelength limit the squeezing mode is very soft. This is the reason why one anticipates that fluctuational effects related to the mode are relevant. The effects are associated with nonlinear terms in dynamic equations and can be examined in terms of the diagrammatic technique of the type first developed by Wyld [22] who studied velocity fluctuations in a turbulent fluid. In the work [23] the Wyld technique was generalised for a broad class of dynamical systems. A textbook description of the diagram technique can be found in the book by Ma [24]. The diagram technique can be formulated in terms of path integrals as was first suggested by de Dominicis [25] and Janssen [26]. In the framework of this approach apart from conventional dynamic variables one should introduce also auxiliary fields conjugated to the variables. Then dynamical correlation functions of the variables can be presented as functional integrals over both type of fields: conventional and auxiliary. The integrals are taken with the weight $\exp(i\mathcal{I})$, where $\mathcal{I}$ is an effective action which is constructed on the basis of nonlinear dynamic equations of the system.

Being interested in the renormalization of the squeezing mode of the double membrane film we will take into account only the variables h and v_α and the corresponding auxiliary conjugated fields p and μ_α. We should also remember about the incompressibility condition (2.3) and impose the analogous constraint $\nabla_\alpha \mu_\alpha = 0$ on the field μ_α. Then, say, the correlation function of the film thickness h is written as

$$\langle h_1 h_2 \rangle = \int \mathcal{D}h\, \mathcal{D}\boldsymbol{v}_{\rm tr} \mathcal{D}p\, \mathcal{D}\boldsymbol{\mu}_{\rm tr} \exp\left(i\mathcal{I}\right) h_1 h_2 \,, \tag{3.1}$$

where the subscript 'tr' implies that in the Fourier representation we should take only transverse to the wave vector $\boldsymbol{q}$ components of the fields $\boldsymbol{v}$ and $\boldsymbol{\mu}$. The explicit expression for the effective action figuring in (3.1) can be found using the dynamical equations (2.5,2.9). It can be written as the sum of the reactive and the dissipative parts $\mathcal{I} = \mathcal{I}_{\rm reac} + \mathcal{I}_{\rm diss}$ where

$$\mathcal{I}_{\rm reac} = \int dt\, d^2r \, \left\{ p\partial_t h + p v_\alpha \nabla_\alpha h + \mu_\alpha \partial_t j_\alpha - \frac{\kappa}{2} \mu_\alpha \nabla^4 h \nabla_\alpha h \right\} , \tag{3.2}$$

$$\mathcal{I}_{\rm diss} = \int dt\, d^2r \, \left\{ -\frac{1}{2}\Gamma\kappa p \nabla^6 h + iT\Gamma (\nabla p)^2 + 2\eta \boldsymbol{\mu} \hat{q} (\boldsymbol{v} + iT\boldsymbol{\mu}) \right\} . \tag{3.3}$$

The detailed derivation of the effective action for the problem can be found in [20,21].

Let us introduce the designations for the pair correlation functions. Taking into account only the transverse components of the fields $\boldsymbol{v}$ and $\boldsymbol{\mu}$ we can write

$$\begin{aligned}
\langle h(t,\boldsymbol{r}) p(0,\boldsymbol{0}) \rangle &= \int \frac{d\omega\, d^2q}{(2\pi)^3} \exp(-i\omega t + i\boldsymbol{q}\boldsymbol{r}) G(\omega,\boldsymbol{q}) \,, \\
\langle v_\alpha(t,\boldsymbol{r}) \mu_\beta(0,\boldsymbol{0}) \rangle &= \int \frac{d\omega\, d^2q}{(2\pi)^3} \exp(-i\omega t + i\boldsymbol{q}\boldsymbol{r}) \left[\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right] G_{\rm tr}(\omega,\boldsymbol{q}) \,, \qquad (3.4) \\
\langle h(t,\boldsymbol{r}) h(0,\boldsymbol{0}) \rangle &= \int \frac{d\omega\, d^2q}{(2\pi)^3} \exp(-i\omega t + i\boldsymbol{q}\boldsymbol{r}) D(\omega,\boldsymbol{q}) \,, \\
\langle v_\alpha(t,\boldsymbol{r}) v_\beta(0,\boldsymbol{0}) \rangle &= \int \frac{d\omega\, d^2q}{(2\pi)^3} \exp(-i\omega t + i\boldsymbol{q}\boldsymbol{r}) \left[\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right] D_{\rm tr}(\omega,\boldsymbol{q}) \,. \qquad (3.5)
\end{aligned}$$

The correlation functions $\langle pp \rangle$ and $\langle \mu\mu \rangle$ are equal to zero (what is the general property of the technique, see e.g. [18]). The functions D and $D_{\rm tr}$ determine the pair correlation functions of the observable quantities and the functions G, $G_{\rm tr}$ are response functions. Therefore, say, the function $G(\omega)$ is analytical in the upper ω half plane.

It is possible to formulate the diagram technique for calculating correlation functions (3.4,3.5). The harmonic part of the effective action $\mathcal{I} = \mathcal{I}_{\rm reac} + \mathcal{I}_{\rm diss}$ determines the 'bare' values of the response functions

$$G_0(\omega,\boldsymbol{q}) = -\frac{1}{\omega + i\Gamma\kappa q^6/2} \,, \qquad G_{{\rm tr},0}(\omega,\boldsymbol{q}) = -\frac{1}{\rho h \omega + 2i\eta q} \,. \tag{3.6}$$

The values of the 'bare' pair correlation functions satisfy the relations

$$Im\, G = \frac{\kappa q^4}{4T} D \,, \qquad Im\, G_{\rm tr} = \frac{1}{2T} D_{\rm tr} \,, \tag{3.7}$$

which are consequences of the fluctuation-dissipation theorem. Besides the harmonic part the effective action $\mathcal{I}$ contains terms of the third order which determine the third-order vertices which figure on diagrams representing the perturbation series for the correlation functions (3.4,3.5). One can check the relations (3.7) order by order and consequently they are true for the 'dressed' correlation functions (3.4,3.5). Note that

$$\int \frac{d\omega}{2\pi} D(\omega, q) = \frac{2T}{\kappa q^4} \,. \tag{3.8}$$

The relation (3.8) is a consequence of (3.7), analyticity of $G(\omega)$ in the upper half-plane and of the asymptotic law $G(\omega) \approx -\omega^{-1}$ which is correct at large ω. Note that (3.8) could be obtained directly from (1.3) since the integral over frequencies is just the simultaneous correlation function.

The analysis of the diagrams shows that they contain infrared logarithms related to the lines representing the correlation function D (3.5). The lines produce factors

$$\langle \nabla_\alpha h(t, \boldsymbol{r}) \nabla_\beta h(t, \boldsymbol{0}) \rangle = \frac{TL}{2\pi\kappa} \delta_{\alpha\beta} \,, \tag{3.9}$$

where $L = \ln[hg^{-1/2}/r]$ and r^{-1} is determined by characteristic external wave vector of the diagram. The expression (3.9) can be found from (3.8) if to remember about the condition (1.5). The presence of the logarithmic contributions implies that the main renormalization of a correlation function like $G(\omega, \boldsymbol{q})$ is produced by the degrees of freedom with the wave vectors much smaller than q. Therefore we should extract from the diagrammatic expressions for $G(\omega, \boldsymbol{q})$ only the contributions corresponding to the interaction with the degrees of freedom.

The program can be realized directly on the language of the functional integral. Let us separate the variables h, p, $\boldsymbol{v}$, $\boldsymbol{\mu}$ into fast parts (with wave vectors larger than q), basic parts (with the wave vectors of the order of q) and slow parts (with wave vectors smaller than q). At calculating $G(\omega, \boldsymbol{q})$ we can forget about the fast parts and keep the interaction of the basic part with the slow part. Then we obtain from (3.2,3.3)

$$\mathcal{I} = \int dt\, d^2r \left\{ p\partial_t h + pv_\alpha m_\alpha + \mu_\alpha \partial_t j_\alpha - \frac{\kappa}{2} \mu_\alpha \nabla^4 h m_\alpha - \Gamma \frac{\kappa}{2} p \nabla^6 h + 2\eta \boldsymbol{\mu} \hat{q} \boldsymbol{v} \right\} + \dots \,, \tag{3.10}$$

where h, p, $\boldsymbol{v}$, $\boldsymbol{\mu}$ denote the basic parts of the fields, m_α is the gradient of the slow part of h and dots designate irrelevant terms. The action (3.10) is of the second order over h, p, $\boldsymbol{v}$, $\boldsymbol{\mu}$ and consequently the integrals over the fields can be taken explicitly. Since $\boldsymbol{m}$ varies only weakly on the length q^{-1} we get

$$G(\omega, \boldsymbol{q}) = -\left\langle (\rho h\omega + 2i\eta q)\Delta^{-1} \right\rangle_m \,, \tag{3.11}$$

$$G_{\rm tr}(\omega, \boldsymbol{q}) = -\left\langle (\omega + i\kappa\Gamma q^6/2)\Delta^{-1} \right\rangle_m \,, \tag{3.12}$$

$$\Delta = (\rho h\omega + 2i\eta q)(\omega + i\kappa\Gamma q^6/2) - \kappa q^4 m_{\rm tr}^2/2 \,, \tag{3.13}$$

where

$$m_{\rm tr}^2 = \left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right) m_\alpha m_\beta \,,$$

and the designation $\langle \dots \rangle_m$ means averaging over statistics of $\boldsymbol{m}$. At calculating (3.11,3.12) we substituted $\boldsymbol{j} = \rho h \boldsymbol{v}$. Actually the terms with ρh can be neglected and we omit the terms below.

At averaging in (3.11,3.12) the statistics of $\boldsymbol{m}$ can be regarded to be Gaussian. The point is that only simultaneous correlation functions of $\boldsymbol{m}$ enter the expressions and the functions are described by the harmonic function (1.3). The pair correlation function of $\boldsymbol{m}$ is equal to (3.9). Therefore

$$\langle m_{\rm tr}^2 \rangle = \frac{TL}{2\pi\kappa} \,,$$

and we find from (3.11)

$$G(\omega, \boldsymbol{q}) = -\int_{-\infty}^{+\infty} \frac{d\zeta}{\sqrt{2\pi}} \exp\left(-\zeta^2/2\right) \left(\omega + i\frac{\kappa\Gamma}{2} q^6 + i\frac{TL}{8\pi\eta} q^3 \zeta^2 \right)^{-1} \,. \tag{3.14}$$

We see that G as a function of the frequency ω have the cut along the imaginary semi-axis which starts from $\omega = -i\Gamma\kappa q^6/2$ and goes up to $-i\infty$. The effective length of the cut can be estimated as Tq^3/η what is the new

characteristic frequency related to fluctuations. Let us compare the frequency with the position of the pole in the bare expression

$$\frac{Tq^3/\eta}{\Gamma\kappa q^6} \sim \frac{g}{(qh)^3}\,. \tag{3.15}$$

We conclude that fluctuation effects dominate in the region $g^{1/2} < qh < g^{1/3}$. Now, we can justify neglecting $\rho h\omega$ in comparison with ηq in the above expressions. At $qh \sim 1$

$$\rho h\omega/(\eta q) \sim \rho\kappa/(\eta^2 h) \sim a/h \ll 1\,,$$

and at $qh \sim \sqrt{g}$

$$\rho h\omega/(\eta q) \sim \rho\kappa/(\eta^2 h)g^2 \ll 1\,.$$

Performing Fourier transform of (3.14) over frequencies one gets

$$G(t,\boldsymbol{q}) = i\left(1+\frac{TL}{4\pi\eta}q^3 t\right)^{-1/2}\exp\left\{-\frac{\kappa}{2}\Gamma q^6 t\right\}\,. \tag{3.16}$$

The expression (3.16) is correct for positive time t, for negative times $G(t) = 0$ due to the causality principle since G is the response function. We see from (3.16) that in the fluctuation region $g^{1/2} < qh < g^{1/3}$ there appears an intermediate power asymptotics $t^{-1/2}$ which at large times t is changed by the exponential decay. That means that the squeezing mode is described by a nonlocal in time dynamic equation.

The above assertion is correct for the wave vectors $q \gtrsim \sqrt{g}\,/h$. In the limit $qh \ll \sqrt{g}$ we return to the local equation (2.5) but with the renormalised kinetic coefficient $\tilde{\Gamma}$. The quantity can be found if to integrate the weight $\exp(i\mathcal{I})$ over the degrees of freedom with the wave vectors $q \gtrsim \sqrt{g}\,/h$. The main effect is related to the sweeping term in the effective action (3.2). Due to the integration over the degrees of freedom with the wave vectors $q \gtrsim \sqrt{g}\,/h$ the term $iT\Gamma(\nabla p)^2$ in (3.3) for the long-wavelength degrees of freedom is renormalised and we found for the renormalised value of Γ:

$$\tilde{\Gamma} - \Gamma = \frac{1}{4T}\int dt\, d^2r\, \langle \boldsymbol{v}(t,\boldsymbol{r})h(t,\boldsymbol{r})\boldsymbol{v}(0,\boldsymbol{0})h(0,\boldsymbol{0})\rangle\,, \tag{3.17}$$

where averaging is performed over the degrees of freedom with the wave vectors $q \gtrsim \sqrt{g}\,/h$. Using the renormalised expressions for the correlation functions we get the estimate $\tilde{\Gamma} \sim g^{-1/2}\Gamma \gg \Gamma$.

IV. CONCLUSION

We demonstrated that fluctuations essentially modify the character of the squeezing mode due to its nonlinear coupling with transversal shear hydrodynamic mode. Fluctuation effects lead to non locality of the equation for the mode, the corresponding Green function is (3.16). The new characteristic frequency of the mode related to fluctuations is $\omega \sim Tq^3/\eta$ (q is the wave vector) which remarkably does not depend on bending elasticity. It is important to distinguish the characteristic frequency from the attenuation of the membrane bending mode (2.2) having the same q^3 dependence on the wave vector. Let us stress that the strong fluctuation effects are observed only for dynamics. Static characteristics are not influenced by fluctuations because of the smallness of the coupling constant (1.2). That is the reason why only the harmonic part of the energy (1.3) is needed for us.

Strong dynamic fluctuations of h occur for the wave vectors $q \gtrsim \sqrt{g}\,/h$. For smaller wave vectors fluctuations of h are weak. Nevertheless even for the wave vectors there is a memory of the region of strong fluctuations which is the renormalised value of the kinetic coefficient Γ in the equation (2.5): The 'bare' value (2.6) is substituted by $\tilde{\Gamma} \sim g^{-1/2}\Gamma \gg \Gamma$. Note also that to analyse the dispersion law of the squeezing mode in the limit $q \ll \sqrt{g}\,/h$ starting from (2.5) one should take into account besides the energy (1.5) also the steric (1.4) and the Van-der-Waals (1.6) contributions to the energy. As a result one finds

$$\omega = -i\tilde{\Gamma}q^2\left(\frac{9\pi^2T^2}{64\kappa h^4} + \frac{10Ha^2}{\pi h^6}\right)\,.$$

Let us discuss possibilities to check our predictions experimentally. The membranes can be studied by a variety of experimental techniques. Lately laser "tweezers" are becoming a useful tool for probing dynamical properties of

membranes. The technique enables one to obtain direct information about amplitudes and characteristic times of dynamical fluctuations for different objects constituted from membranes. For details see the monography [27] and recent experiments [29–31]. We can mention as well force apparatus measurements [28] making possible to investigate dynamical response for two very thin lamellar systems confined between the walls and the classical light scattering experiments. Because of relaxation of membrane fluctuations, the scattered light has a broadened spectral distribution compared with the incident light. Despite the broadening is small, the modern technique of light beating (intensity fluctuation spectroscopy) is allowed to obtain information about eigen-modes of the system.

The conclusions concerning the renormalization of the squeezing mode in our opinion are interesting both in its own right and as a new test of the membrane fluctuations.

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